

# Cascade Failures from Distributed Generation in Power Grids

Antonio Scala<sup>1,2,3</sup>, Sakshi Pahwa<sup>4</sup>, and  
Caterina Scoglio<sup>4</sup>

<sup>1</sup> ISC-CNR Physics Dept., Univ. "La Sapienza" Piazzale Moro 5, 00185 Roma, Italy

<sup>2</sup> IMT Alti Studi Lucca, piazza S. Ponziano 6, 55100 Lucca, Italy

<sup>3</sup> London Institute of Mathematical Sciences, 22 South Audley St  
Mayfair London W1K 2NY, UK

<sup>4</sup> Department of Electrical and Computer Engineering, College of Engineering  
Kansas State University, Manhattan, KS

**Abstract.** Power grids are nowadays experiencing a transformation due to the introduction of Distributed Generation based on Renewable Sources. At difference with classical Distributed Generation, where local power sources mitigate anomalous user consumption peaks, Renewable Sources introduce in the grid intrinsically erratic power inputs. By introducing a simple schematic (but realistic) model for power grids with stochastic distributed generation, we study the effects of erratic sources on the robustness of several IEEE power grid test networks with up to  $2 \times 10^3$  buses. We find that increasing the penetration of erratic sources causes the grid to fail with a sharp transition. We compare such results with the case of failures caused by the natural increasing power demand.

**Keywords:** distributed generation, DC power model, cascading failures, first-order transition

## 1 Introduction

The unavailability of massive and economic power storage does not allow yet to integrate the stochastic and often volatile renewable sources by leveraging on distributed storage [3]. The power-on-demand paradigm upon which power grids have been originally engineered did not contemplate the introduction of Distributed Generation from renewable sources; at the same time, power grids are nowadays required to be robust and smart, i.e. to be able to maintain, under normal or perturbed conditions, the frequency and amplitude supply voltage variations into a defined range and to provide fast restoration after faults.

In order to understand how to ensure stability and avoid loss of synchronization, most studies have concentrated on the dynamic behavior of Smart Grids during typical events like the interconnection of distributed generation. To study such events in grids with a large number of elements, simplifications like the mapping among the classic swing equations [19] and Kuramoto models [11,12,9] are welcome in simplifying the numerical and the analytical study of the synchronization and the transient stability of a power network.

Beside dynamic instabilities, new kind of failures are possible as Smart grids are going to insist on pre-existing networks designed for different purposes and tailored on different paradigms and : simple models [8] akin to the DC power flow model [21] show that the network topology can dynamically induce complex blackout size probability distributions (power-law distributed), both when the system is operated near its limits [6] or when the system is subject to erratic disturbances [18]. In general, new realistic metrics to assess the robustness of the electric power grid with respect to complex events like cascading failures [22] are needed.

A further effect of the introduction of Distributed Generation from renewable sources is the possibility of an erratic reshaping of the magnitude and direction power flows. We will focus instead on the condition under which, in presence of distributed generation, the system can be brought beyond its design parameters and subject to cascading failures.

To model distributed renewable sources, we will introduced a probability distribution of load demands representing fluctuations due to consumer demand and to an erratic distributed generation. Our model is a crude model of reality that ignores effects like the correlations between different consumers or distributed producers (due for examples to weather conditions) or time correlations (like the time-cyclic components of the demand).

To sample the effects of stochastic demand and production, we will employ intensive Monte Carlo methods in which possible power flows configurations are repeatedly calculated during cascading events; therefore, we will use the less computational demanding DC power flow model as the building block of our simulations. We will then analyze the redistribution of flows due to branch failures and the subsequent fragmenting of the power grids caused by overloading cascades.

## 2 Methods

### 2.1 Power Flow models

The AC power flow is described by a system of non-linear equations that allow to obtain complete voltage angle and magnitude information for each bus in a power system for specified loads [13]. A bus of the system is either classified as Load Bus if there are no generators connected or as a Generator Bus if one or more generators are connected. It is assumed that the real power  $PD$  and the reactive power  $QD$  at each Load Bus are given while for Generator Buses the real generated power  $PG$  and the voltage magnitude  $|V|$  are given. A particular Generator Bus, called the Slack Bus, is assumed as a reference and its voltage magnitude  $|V|$  and voltage phase  $\Theta$  are fixed. The branches of the electrical system are described by the bus admittance matrix  $Y$ .

The power balance equations can be written for real and reactive power for each bus; real and reactive power flow on each branch as well as generator reactive power output can be analytically determined, but due to the non-linear character of the system numerical methods are employed to obtain a solution.

A simplification of the AC power flow equations is obtained by linearizing the equations by requiring that bus voltages  $V_i$  are fixed and phase differences  $\theta_{ij} = \Theta_i - \Theta_j$  along the branches are small. The resulting linear system of equations constitutes the DC power-flow model [14]

$$\mathbf{P} = \mathcal{L}\mathbf{\Theta} \quad (1)$$

where  $P_i$  is the total power (generation minus load) at the  $i$ -th bus,  $\mathcal{L} = K - Y$  is the Laplacian matrix with  $K$  diagonal degree matrix  $K_{ii} = \sum_j Y_{ij}$ ; the sign of  $P_i$  determines if a node is a generator, a load or even a transit node ( $P_i = 0$ ). Notice that we are neglecting phase shifts of the transformers that would add an extra term in eq.(1):  $\mathbf{P} = \mathcal{L}\mathbf{\Theta} + \mathbf{P}^\phi$ .

## 2.2 Overload Cascade model

To consider the effects of customer behavior and of the distributed generation due to erratic renewable sources like sun and wind in the simplest approximation, we model the effects of “green generators” and of variable customer demand as a stochastic variation of the power requested by load buses.

We will assume the case of the full penetration of renewable sources in the grid; therefore, all the loads will be considered random variables.

A requirement for the stability of the load and generation is the condition that all branches and buses operate within their physical feasibility parameters; going beyond such parameters can trigger cascades of failures eventually leading to black outs [17].

The redistribution of power is dependent on the electrical characteristics, such as impedances, of transmission lines. We neglect line resistances because they are very small as compared to their inductive reactances [14].

Flow in power grids has a complex dynamics even in the DC approximation: if a single branch gets overloaded or breaks, its power is immediately distributed not to a single different branch but in the whole system. If the redistribution of power leads to the subsequent overloading of other branches, the consequence could be a cascade of overloading failures. In fact, after an initial failure, some of the branches can get overloaded and fail: this represents the first stage of cascade. First stage could possibly lead to overloading and collapse of further branches, constituting the second stage and so on. In this way, the system goes through multiple stages of cascade until it finally stabilizes and there are no more failures. We indicate the final stable configuration by  $\{y\}$ , where

$$y_{ij} = \begin{cases} 0 & \text{if branch } (i, j) \text{ is broken} \\ Y_{ij} & \text{otherwise} \end{cases} \quad (2)$$

and  $|y_{ij}\theta_{ij}| < F_{ij}$ , where  $F_{ij}$  is the threshold flow beyond which a branch becomes overloaded and fails.

The Overload Cascade Model (OCM) therefore corresponds to a double minimization of the objective function

$$H(\{y\}) = \sum_{ij} y_{ij}^2 [\theta_{ij}^2 - T_{ij}^2] \quad (3)$$

(here  $T_{ij} = F_{ij}/Y_{ij}$  and  $y_{ij} \in \{0, Y_{ij}\}$ ), both respect to global variables  $\theta_{ij}$  at fixed  $y_{ij}$  (via DC power flow) and respect to the local variables  $y_{ij}$  at fixed  $\theta_{ij}$  (breakdown of overloaded links). Therefore, the OCM can be mapped in the model for the breakdown of a disordered media [23], indicating that the cascading transition in power grids is a first order transition, i.e. consists in an abrupt failure of the system.

### 3 Results

#### 3.1 AC-DC power flow comparison

The DC model is often used in applications where fast convergence is needed, like in the case of real-time systems or when trying to optimize a system calculations must be repeated under different conditions. DC power flow is on average wrong by a few percent [20] respect to the more computational intensive AC power flow. In our case with stochastic sources, we want to explore the probability that links get overloaded and fail; therefore, we need a correct statistics of the power dissipated along the branches. To solve both DC and AC power flow equations, we employ the MATPOWER libraries [1] under the scientific environment for numerical calculus Octave [2,10].

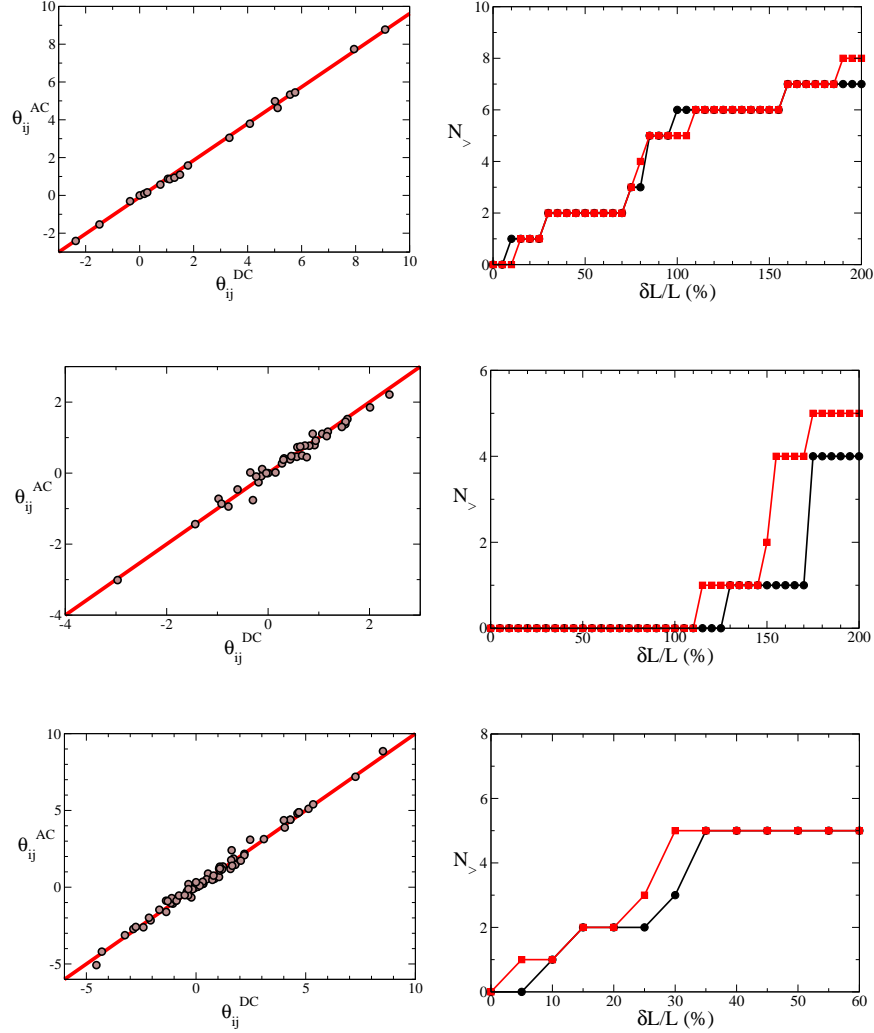
We first compare DC and AC phase shifts  $\theta$  along the branches of several grids; the left panels of fig.(1) shows that the two models produce highly similar results for the flows.

We then compare the number of branches with a high power flux ( $|\theta_{ij}| > 10^\circ$ ) when incrementing/decrementing the loads respect to the initial conditions; the right panel of fig.(1) shows that both the DC and the AC model consistently produce an equivalent number of branches above the threshold.

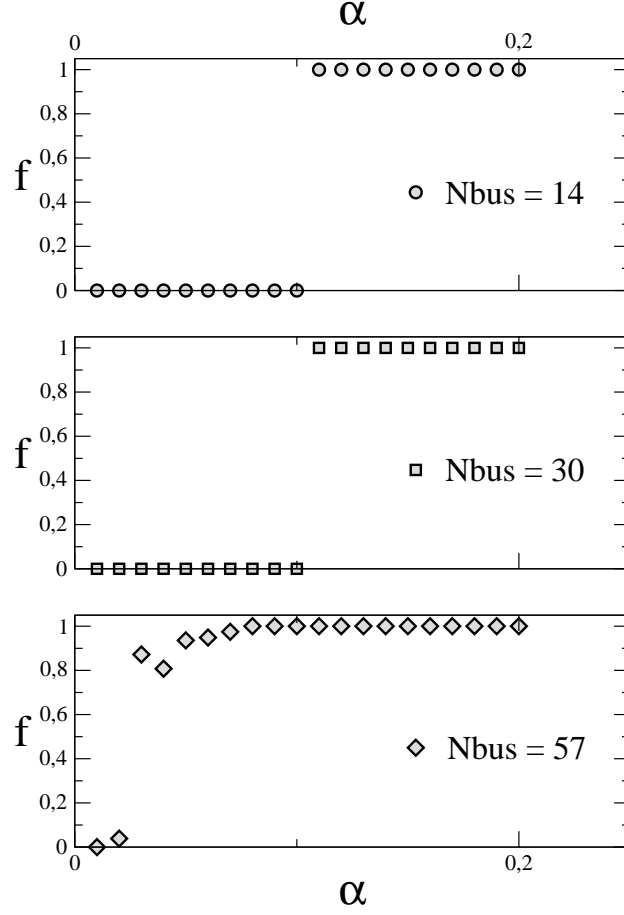
Notice that the number of high power flux branches increases also when loads are decreased; this is an indication of the fact that power grids are designed to work under certain flow configurations and therefore distributed generation can introduce instabilities and line overloads.

#### 3.2 Cascades from Correlated Overload

Customer demand is on average increasing with time, so every kind of human-built network will eventually reach its capacity limit beyond which it becomes dis-functional or even breaks down. To mimic such events, we analyze via the OCM the effects of a correlated load increase on several model power grids. The loads  $L_i$  on the  $i$ -th bus are increased uniformly by a factor  $1 + \alpha$ ; in such a situation the mean-field model of [23] predicts a first order transition (an abrupt jump) in the system. Figure (2) shows that this is indeed the case; notice that up to the transition the network stays virtually intact, signaling a very robust design against load increases. On the other hand, the fact that in power grids interactions are non-local (due to the non-local character of power equations) will eventually lead to a systemic failure for high enough stresses even for very well conceived power grids.



**Fig. 1.** Comparison of DC and AC phase shifts in the 14, the 30 and the 57 bus model grids (top to bottom). Left panels: The phase angle differences for AC and DC power flows are highly correlated  $\theta_{ij}^{AC} \sim \theta_{ij}^{DC}$ , as seen with the comparison with the line  $y = x$  (thick straight line). Right panels: Variation of the number  $N_{>}$  of branches with phase shift above the threshold  $|\theta_{ij}| > 10^\circ$  (high flow of power) while varying uniformly the loads by a factor  $\delta L/L$  respect to the initial condition (where  $L$  is the initial total load). Again, the behavior is equivalent for the DC and the AC models. Notice that the biggest grid (57 buses) becomes unstable even for smaller loads  $\delta L/L < 0$  (not shown in figure), indicating that it is designed to work around a given flow configuration.



**Fig. 2.** Comparison of the cascade damage from correlated overload for the 14, the 30 and the 57 bus model grids (top to bottom). Loads are simultaneously increased by a factor  $1 + \alpha$  (i.e.  $\alpha = 0.2$  corresponds to a 20% increase of the loads); notice that the fraction  $f$  of damaged links suffers an abrupt jump as predicted from the mean-field theory.

### 3.3 Effects of distributed generation

Distributed generation from renewable sources and customer behavior are both sources of randomness in the power demand/production of a grid. It is therefore important to understand the robustness of grids not only respect to variations of the total demand/production but also due to their local (i.e. on single buses) fluctuations. To model such issues in the simplest way, we consider the load  $L_i$  on the  $i$ -th bus to be a random variable uniformly distributed in the interval  $[L_i^0(1 - \sigma), L_i^0(1 + \sigma)]$  where  $L_i^0$  is the initial load and  $\sigma$  measures the strength of the fluctuations; therefore for  $\sigma = 1$  each load bus can fluctuate by 100% of its initial value

It is well known that randomness can eventually smooth out transitions [15]; we find that even for the OCM model for random fluctuation transitions are smoother than the case of correlated loads. Nevertheless, fig.(3) also shows that transitions get steeper when increasing the number of buses as expected from the mean-field character of the OCM.

## 4 Discussion

We have introduced a model of overload failure cascades based on the DC power flow equation that allows to account for the presence of erratic renewable sources distributed on a power grid.

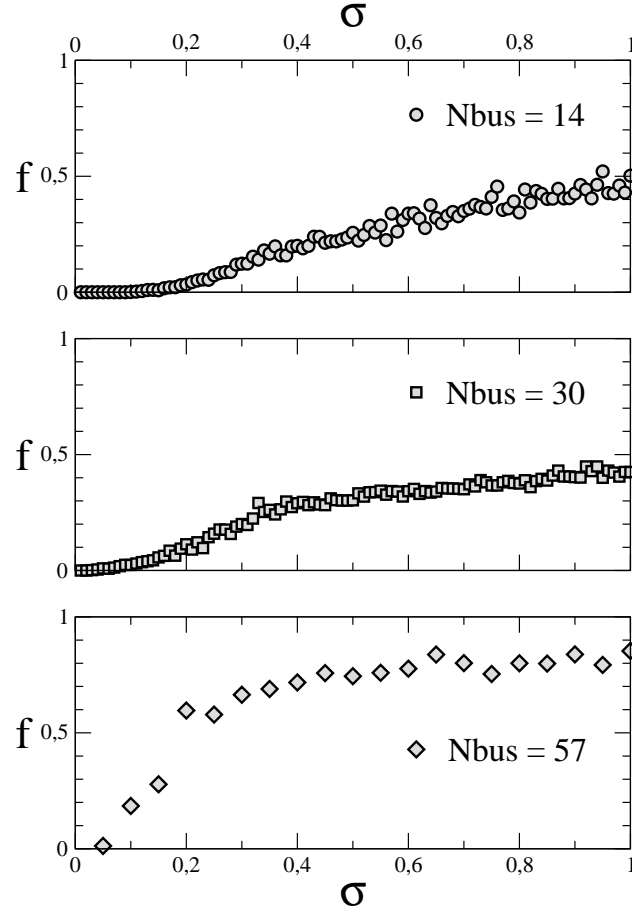
While we found that fluctuations increase the instability within an isolated grid, what happens when more grids are linked together is an open subject. Power grids are typical complex infrastructural systems; therefore they can exhibit emergent characteristics when they interact with each other, modifying the risk of failure in the individual systems [7]. As an example, the increase in infrastructural interdependencies could either mitigate [4] or increase [16,5] the risk of a system failure.

We find that an increasing uniform stress in power grids can induce abrupt failures consistently with the universality class of the model of cascade failures we have introduced. On the other hand, the presence of fluctuations due to erratic renewable sources or to customer demand induces an apparently smoother breakdown of the grid; whether such transition becomes steeper for bigger grids (as indicated from our simulations and hinted from the mean-field character of our cascade model) is an important point to assess in the near future.

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**Fig. 3.** Comparison of the cascade damage from random overloads for the 14, the 30 and the 57 bus model grids (top to bottom). The load  $L_i$  on each node  $L_i$  is subject to a uniform fluctuation between  $L_i \pm \sigma L_i$  modeling erratic behavior from distributed renewable power sources and customer behavior. The fraction  $f$  of damaged links has a smoother behavior when compared to the correlated load increase case; nevertheless, by increasing the number of nodes in the grid, the transition to the failed states becomes steeper, consistently with the first-order transition picture of the Overload Cascade Model.



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